Finding sine given cosine, or vice versa

As you may have already figured out, if you're given the cosine of some angle θ, you can easily find the value of sine using the definitions of $\sin\theta$ and $\cos\theta$:

$$\sin\theta = y$$

$$\cos\theta = x$$

where y and x are the y- and x-coordinates of points along the unit circle. You also need to remember that the equation of the unit circle is

$$x^2 + y^2 = 1$$

Combining the definitions of $\sin\theta$ and $\cos\theta$ with the equation of the unit circle, we have

$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

$$\cos^2\theta + \sin^2\theta = 1$$

So if we know the value of $\cos\theta$ and we want to find $\sin\theta$, we can use

$$\sin^2\theta = 1 - \cos^2\theta$$

And if we know the value of $\sin\theta$ and we want to find $\cos\theta$, we can use the equation

$$\cos^2\theta = 1 - \sin^2\theta$$

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When we try to solve either one of these, we'll see that we always get positive and negative options. For example, if we solve the first equation for $\sin\theta$, we get

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sqrt{\sin^2\theta} = \sqrt{1 - \cos^2\theta}$$

$$\sin\theta = \pm\sqrt{1 - \cos^2\theta}$$

The same $\pm$ thing will happen with the cosine equation. In order to figure out whether we want the positive value or the negative value, we just need the quadrant of the angle, which will tell us the sign. Remember

In Quadrant I, $y = \sin\theta = +$ and $x = \cos\theta = +$

In Quadrant II, $y = \sin\theta = +$ and $x = \cos\theta = -$

In Quadrant III, $y = \sin\theta = -$ and $x = \cos\theta = -$

In Quadrant IV, $y = \sin\theta = -$ and $x = \cos\theta = +$

Example

If the value of $\cos(236^\circ)$ is -0.559, what's the value of $\sin(236^\circ)$?

We'll use the rewritten form $\sin^2\theta = 1 - \cos^2\theta$ of the basic Pythagorean identity to write

$$\sin^2(236^\circ) = 1 - \cos^2(236^\circ)$$

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Substituting the value of $\cos(236°)$, we get

$$\sin^2(236°) = 1 - (-0.559)^2$$

$$\sin^2(236°) \approx 1 - 0.312$$

$$\sin^2(236°) \approx 0.688$$

$$\sin(236°) \approx \pm\sqrt{0.688}$$

The angle $236°$ lies in the third quadrant, and all points in the third quadrant have a negative y-value, which means that the sine of any angle in the third quadrant will be negative. So we ignore the positive value and say

$$\sin(236°) \approx -\sqrt{0.688}$$

$$\sin(236°) \approx -0.829$$

We can use the same process procedure to get the value of $\cos\theta$ from the value of $\sin\theta$.

Example

Find $\cos(49.3°)$ if $\sin(49.3°) = 0.758$.

We'll use the rewritten form $\cos^2\theta = 1 - \sin^2\theta$ of the basic Pythagorean identity to write

$$\cos^2(49.3°) = 1 - \sin^2(49.3°)$$

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Substituting the value of $\sin(49.3°)$, we get

$$\cos^2(49.3°) = 1 - (0.758)^2$$

$$\cos^2(49.3°) \approx 1 - 0.575$$

$$\cos^2(49.3°) \approx 0.425$$

$$\cos(49.3°) \approx \pm\sqrt{0.425}$$

The angle $49.3°$ lies in the first quadrant, and all points in the first quadrant have a positive x-value, which means that the cosine of any angle in the first quadrant will be positive. So we ignore the negative value and say

$$\cos(49.3°) \approx \sqrt{0.425}$$

$$\cos(49.3°) \approx 0.652$$

Let’s look at an example with a negative angle.

Example

What's the value of $\cos(-152°)$ if $\sin(-152°) = -0.469$?

Here, we'll use the equation

$$\cos^2(-152°) = 1 - \sin^2(-152°)$$

Substituting the value of $\sin(-152°)$, we have

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$$\cos^2(-152^\circ) = 1 - (-0.469)^2$$

$$\cos^2(-152^\circ) \approx 1 - 0.220$$

$$\cos^2(-152^\circ) \approx 0.780$$

$$\cos(-152^\circ) \approx \pm\sqrt{0.780}$$

The angle -152° lies in the third quadrant, and all points in the third quadrant have a negative x-value, which means that the cosine of any angle in the third quadrant will be negative. So we ignore the positive value and say

$$\cos(-152^\circ) \approx -\sqrt{0.780}$$

$$\cos(-152^\circ) \approx -0.883$$

Let's do an example with an angle in radians.

Example

Let $\theta = -11\pi/9$. If $\cos\theta = -0.766$, what's the value of $\sin\theta$?

Using the basic Pythagorean identity, we can write

$$\sin^2\left(-\frac{11\pi}{9}\right) = 1 - \cos^2\left(-\frac{11\pi}{9}\right)$$

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$$\sin^2\left(-\frac{11\pi}{9}\right) = 1 - (-0.766)^2$$

$$\sin^2\left(-\frac{11\pi}{9}\right) \approx 1 - 0.587$$

$$\sin^2\left(-\frac{11\pi}{9}\right) \approx 0.413$$

$$\sin\left(-\frac{11\pi}{9}\right) \approx \pm\sqrt{0.413}$$

The angle $-11\pi/9$ lies in the second quadrant, and all points in the second quadrant have a positive y-value, which means that the sine of any angle in the second quadrant will be positive. So we ignore the negative value and say

$$\sin\left(-\frac{11\pi}{9}\right) \approx \sqrt{0.413}$$

$$\sin\left(-\frac{11\pi}{9}\right) \approx 0.643$$

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